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Design of Concrete Mixtures

By

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RESEARCHES in the properties of concrete and concrete materials at the Structural Materials Research Laboratory are being carried out through the cooperation of the Lewis Institute and the Portland Cement Association, Chicago. The research work has been under way since September 1, 1914.

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Design of Concrete Mixtures*

By DUFF A. ABRAMS

PROFESSOR IN CHARGE OF LABORATORY

The design of concrete mixtures is a subject of vital interest to all engineers and constructors who have to do with concrete work. The problem involved may be one of the following:

1. What mix is necessary to produce concrete of proper strength for a given work?
2. With given materials what proportions will give the best concrete at minimum cost?
3. With different lots of materials of different characteristics which is best suited for the purpose?
4. What is the effect on strength of concrete from changes in mix, consistency or size and grading of aggregate?

Proportioning concrete frequently involves selection of materials as well as their combination. In general, the question of relative costs is also present.

The term "Design" is used in the title of this article as distinguished from "proportioning" since it is the intention to imply that each element of the problem is approached with a deliberate purpose in view which is guided by a rational method of accomplishment.

The design of concrete mixtures, with a view to producing a given result in the most economic manner, involves many complications which have heretofore defied analysis.

Many different methods of proportioning have been suggested; the most important ones may be characterized as follows:

1. Arbitrary selection, such as 1:2:4 mix, without reference to the size or grading of the fine and coarse aggregate;
2. Density of aggregates in which the endeavor is made to secure an aggregate of maximum density;
3. Density of concrete in which the attempt is made to secure concrete of maximum density;
4. Sieve analysis, in which the grading of the aggregates is made to approximate some predetermined sieve analysis curve which is considered to give the best results;
5. Surface area of aggregates.

It is a matter of common experience that the method of arbitrary selection in which fixed quantities of fine and coarse aggregates are mixed without regard to the size and grading of the individual materials, is far from satisfactory. Our experiments have shown that the other methods mentioned above are also subject to serious limitations. We have found that the maximum strength of concrete does not depend on either an aggregate of maximum density or a concrete of maximum density, and that the methods which have been suggested for proportioning concrete by sieve analysis of aggregates are based on an erroneous theory. All of the methods of proportioning concrete which have been proposed in the past have failed to give proper attention to the water content of the mix. Our experimental work has emphasized the importance of the water in concrete mixtures, and shown that the water is, in fact, the most important ingredient, since very small variations in water content produce more important variations in the strength and other properties of concrete than similar changes in the other ingredients.

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*Reprinted from Minutes of the Annual Meeting of the Portland Cement Association, held in New York, December, 1918.

New Studies of Concrete Mixtures

During the past three years a large number of investigations have been under way at the Structural Materials Research Laboratory, Lewis Institute, Chicago, which throw considerable new light on the subject of proportioning concrete. These investigations are being carried out through the cooperation of the Institute and the Portland Cement Association. These studies have covered an investigation of the inter-relation of the following factors:

1. The consistency (quantity of mixing water).
2. The size and grading of aggregates.
3. The mix (proportion of cement).

Any comprehensive study of proportioning concrete must take into account all of these factors.

During this period about 50,000 tests have been carried out which have a bearing on this subject. These tests have been largely confined to compression tests of concrete and mortars. These investigations have given us a new insight into the factors which underlie the correct proportioning of concrete mixtures and show the limitations of older methods. Certain phases of these investigations are still under way.

The following may be mentioned as among the most important principles which have been established with reference to the design of concrete mixtures. In a brief report of this kind it is impracticable to present more than an outline of the methods of applying the principles to practical problems. In only a few instances are experimental data given on which these conclusions are based.

1. With given concrete materials and conditions of test the quantity of mixing water used determines the strength of the concrete, so long as the mix is of a workable plasticity.
2. The sieve analysis furnishes the only correct basis for proportioning aggregates in concrete mixtures.
3. A simple method of measuring the effective size and grading of an aggregate has been developed. This gives rise to a function known as the "fineness modulus" of the aggregate.
4. The fineness modulus of the aggregate furnishes a rational method for combining materials of different size for concrete mixtures.
5. The sieve analysis curve of the aggregate may be widely different in form without exerting any influence on the concrete strength.
6. Aggregate of equivalent concrete-making qualities may be produced by an infinite number of different gradings of a given material.
7. Aggregates of equivalent concrete-making qualities may be produced from materials of widely different size and grading.
8. In general, fine and coarse aggregates of widely different size or grading can be combined in such a manner as to produce similar results in concrete.
9. The aggregate grading which produces the strongest concrete is not that giving the maximum density (lowest voids). A grading coarser than that giving maximum density is necessary for highest concrete strength.
10. The richer the mix, the coarser the grading should be for an aggregate of given maximum size; hence, the greater the discrepancy between maximum density and best grading.
11. A complete analysis has been made of the water-requirements of concrete mixes. The quantity of water required is governed by the following factors:
    (a) The condition of "workability" of concrete which must be used—the relative plasticity or consistency;
    (b) The normal consistency of the cement;
    (c) The size and grading of the aggregate—measured by the fineness modulus;

(d) The relative volumes of cement and aggregate—the mix;
(e) The absorption of the aggregate;
(f) The contained water in aggregate.

12. There is an intimate relation between the grading of the aggregate and the quantity of water required to produce a workable concrete.

13. The water content of a concrete mix is best considered in terms of the volume of the cement—the water-ratio.

14. The shape of the particle and the quality of the aggregate have less influence on the concrete strength than has been reported by other experimenters.

Effect of Quantity of Mixing Water on the Strength of Concrete

Fig. 1 shows the relation between the compressive strength and the water content for 28-day tests of 6 by 12-in. concrete cylinders. Mixes from 1:15 to neat cement were used; each mix was made up of aggregates ranging in size from 14-mesh sand up to 1½-in. gravel; a wide range in consistencies was used for all mixes and gradings.

The water content of the concrete is expressed as a ratio to the volume of cement, considering that the cement weighs 94 lb. per cu. ft. Distinguishing marks are used for each mix, but no distinction is made between aggregates of different size or different consistencies.

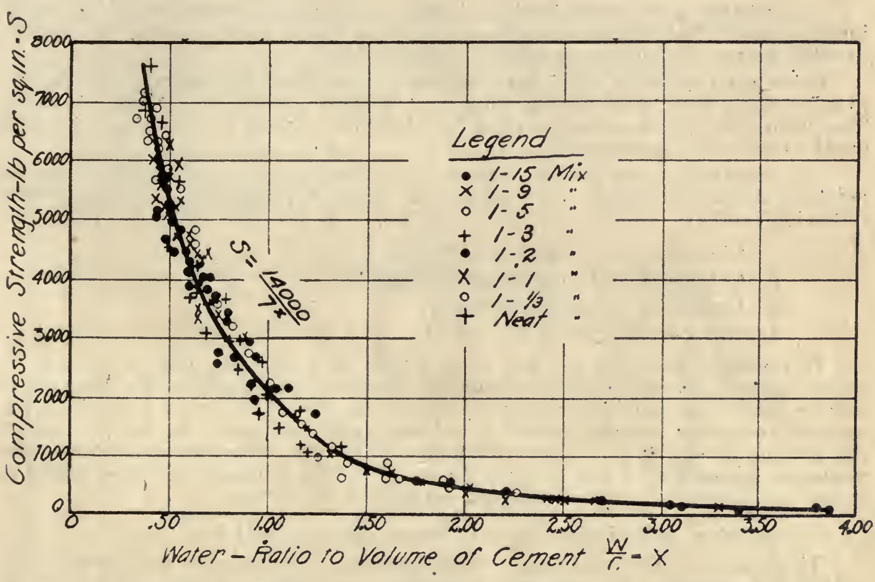

FIG. 1. RELATION BETWEEN STRENGTH OF CONCRETE AND WATER CONTENT
Twenty-eight-day compression tests of 6 by 12-inch cylinders. (Series 83.)

When the compressive strength is platted against the water ratio in this way, a smooth curve is obtained, due to the overlapping of the points for different mixes. Values from dry concretes have been omitted. If these were used we should obtain a series of curves dropping downward and to the left from the curve shown. It is seen at once that the size and grading of the aggregate and the quantity of cement are no longer of any importance except in so far as these factors influence the quantity of water required to produce a workable mix. This gives us an entirely new conception of the function of the constituent materials entering into a concrete mix and is the most basic principle which has been brought out in our studies of concrete.

The equation of the curve is of the form,

$$S=\frac{A}{B^x}\ldots\ldots(1)$$

where S is the compressive strength of concrete and x is the ratio of the volume of water to the volume of cement in the batch. A and B are constants whose values depend on the quality of the cement used, the age of the concrete, curing conditions, etc.

This equation expresses the law of strength of concrete so far as the proportions of materials are concerned. It is seen that for given concrete materials the strength depends on only one factor—the ratio of water to cement. Equations which have been proposed in the past for this purpose contain terms which take into account such factors as quantity of cement, proportions of fine and coarse aggregate, voids in aggregate, etc., but they have uniformly omitted the only term which is of any importance; that is, *the water*.

For the conditions of these tests, equation (1) becomes,

$$S=\frac{14{,}000}{7^x}\ldots\ldots(2)$$

The relation given above holds so long as the concrete is not too dry for maximum strength and the aggregate not too coarse for a given quantity of cement; in other words, so long as we have a *workable mix*.

Other tests made in this laboratory have shown that the character of the aggregate makes little difference so long as it is clean and not structurally deficient. The absorption of the aggregate must be taken into account if comparison is being made of different aggregates.

The strength of the concrete responds to changes in water, regardless of the reason for these changes. The water-ratio may be changed due to any of the following causes:

1. Change in mix (cement content).
2. Change in size or grading of aggregate.
3. Change in relative consistency.
4. Any combination of (1) to (3).

In certain instances a 1:9 mix is as strong as a 1:2 mix, depending only on water content. It should not be concluded that these tests indicate that lean mixes can be substituted for richer ones without limit. We are always limited by the necessity of using sufficient water to secure a workable mix. So in the case of the grading of aggregates. The workability of the mix will in all cases dictate the minimum quantity of water that can be used. The importance of the workability factor in concrete is therefore brought out in its true relation.

The problem of designing concrete mixes resolves itself into this:

To produce a workable concrete which has a given water-ratio using a minimum quantity of cement; or the converse, to produce a workable concrete with a minimum water-ratio using a given quantity of cement. The methods for securing the best grading of aggregate and the use of the driest concrete which is workable are thus seen to be only devices which enable us to accomplish the above-mentioned results.

Fineness Modulus of Aggregate

The experimental work carried out in the laboratory has given rise to what we term the fineness modulus of the aggregate. This function furnishes a method of measuring the size and grading of the aggregate. It may be defined as follows:

The sum of the percentages in the sieve analysis of the aggregate divided by 100.

The sieve analysis is determined by using the following sieves from the Tyler standard series: 100, 48, 28, 14, 8, 4, ⅜-in., ¾-in. and 1½-in. These sieves are made of square-mesh wire cloth. Each sieve has a clear opening just double the

Table 1

METHOD OF CALCULATING FINENESS MODULUS OF AGGREGATES

The *sieves* used are commonly known as the Tyler standard sieves. Each sieve has a *clear opening* just double that of the preceding one.

The *sieve analysis* may be expressed in terms of volume or weight.

The *fineness modulus* of an aggregate is the sum of the percentages given by the sieve analysis, divided by 100.

Sieve Size	Size of Square Opening in.	Size of Square Opening mm.	Sand Fine (A)	Sand Medium (B)	Sand Coarse (C)	Pebbles Fine (D)	Pebbles Medium (E)	Pebbles Coarse (F)	Concrete Aggregate (G)*
			Sieve Analysis of Aggregates Per Cent of Sample Coarser than a Given Sieve						
100-mesh	.0058	.147	82	91	97	100	100	100	98
48-mesh	.0116	.295	52	70	81	100	100	100	92
28-mesh	.0232	.59	20	46	63	100	100	100	86
14-mesh	.046	1.17	0	24	44	100	100	100	81
8-mesh	.093	2.36	0	10	25	100	100	100	78
4-mesh	.185	4.70	0	0	0	86	95	100	71
⅜-in.	.37	9.4	0	0	0	51	66	86	49
¾-in.	.75	18.8	0	0	0	9	25	50	19
1½-in.	1.5	38.1	0	0	0	0	0	0	0
Fineness Modulus			1.54	2.41	3.10	6.46	6.86	7.36	5.74

*Concrete aggregate "G" is made up of 25% of sand "B" mixed with 75% of pebbles "E." Equivalent gradings would be secured by mixing 33% sand "B" with 67% coarse pebbles "F"; 28% "A" with 72% "F," etc. The proportion coarser than a given sieve is made up by the addition of these percentages of the corresponding size of the constituent materials.

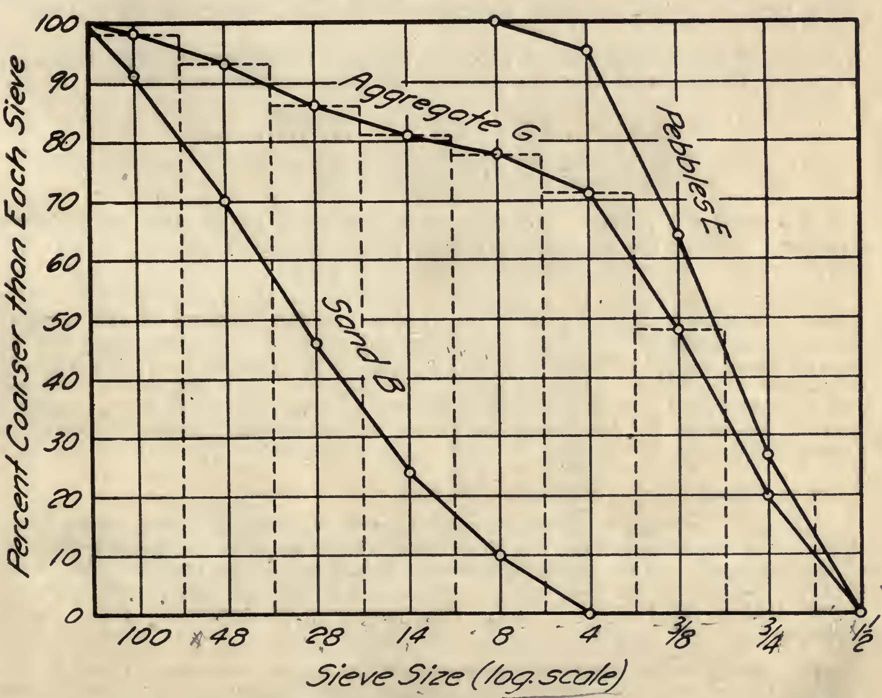

FIG. 2. METHOD OF PLOTTING SIEVE ANALYSIS OF AGGREGATES
Sieve analysis curves for aggregates B, E and G in Table 1.

width of the preceding one. The exact dimensions of the sieves and the method of determining the fineness modulus will be found in Table 1. It will be noted that the sieve analysis is expressed in terms of the percentages of material by volume or weight *coarser than* each sieve.

A well-graded torpedo sand up to No. 4 sieve will give a fineness modulus of about 3.00; a coarse aggregate graded 4-1½ in. will give fineness modulus of about 7.00; a mixture of the above materials in proper proportions for a 1:4 mix will have a fineness modulus of about 5.80. A fine sand such as drift-sand may have a fineness modulus as low as 1.50.

Sieve Analysis of Aggregates

There is an intimate relation between the sieve analysis curve for the aggregate and the fineness modulus; in fact, the fineness modulus enables us for the first time to properly interpret the sieve analysis of an aggregate. If the sieve analysis of an aggregate is platted in the manner indicated in Fig. 2; that is, using the per cent coarser than a given sieve as ordinate, and the sieve size (platted to logarithmic scale) as abscissa, the fineness modulus of the aggregate is measured by the area below the sieve analysis curve. The dotted rectangles for aggregate "G" show how this result is secured. Each elemental rectangle is the fineness modulus of the material of that particular size. The fineness modulus of the graded aggregate is then the summation of these elemental areas. Any other sieve analysis curve which will give the same total area corresponds to the same fineness modulus and will require the same quantity of water to produce a mix of the same plasticity and gives concrete of the same strength, so long as it is not too coarse for the quantity of cement used.

The fineness modulus may be considered as an abstract number; it is in fact a summation of volumes of material. There are several different methods of computing it, all of which will give the same result. The method given in Table 1 is probably the simplest and most direct.

Some of the mathematical relations involved are of interest. The following expression shows the relation between the fineness modulus and the size of the particle:

$$m = 7.94 + 3.32 \log d \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Where m = fineness modulus
d = diameter of particle in inches

This relation is perfectly general so long as we use the standard set of sieves mentioned above. The constants are fixed by the particular sizes of sieves used and the units of measure. Logarithms are to the base 10.

This relation applies to a single-size material or to a given particle. The fineness modulus is then a logarithmic function of the diameter of the particle. This formula need not be used with a graded material, since the value can be secured more easily and directly by the method used in Table 1. It is applicable to graded materials provided the relative quantities of each size are considered, and the diameter of each group is used. By applying the formula to a graded material we would be calculating the values of the separate elemental rectangles shown in Fig. 2.

Fineness Modulus Strength Relation for Concrete

Many different series of tests have shown that for a given plastic condition of concrete and the same mix there is an intimate relation between the fineness modulus of the aggregate and the strength and other properties of the concrete. We have seen that the reason for this result is found in the fact that the fineness modulus simply reflects the changes in water-ratio necessary to produce a given plastic condition.

Figs. 3 and 4 give the results of certain compression tests which bring out the relation between the strength of the concrete and the fineness modulus of the aggregate. It will be noted from Fig. 3 that a separate curve may be drawn for each mix. In each case there is a steady increase in the compressive strength of

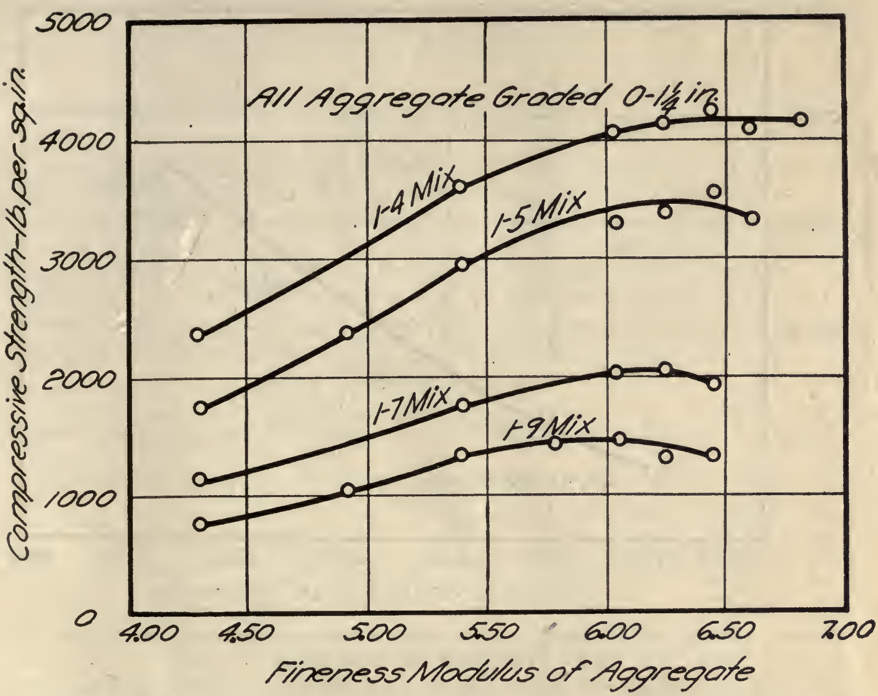

FIG. 3. RELATION BETWEEN FINENESS MODULUS OF AGGREGATE AND STRENGTH OF CONCRETE

Sand and pebble aggregate graded 0-1¼ inch; 28-day compression tests of 6 by 12-inch cylinders. (Series 78.)

The sieve analyses of aggregates are given below:

Range in Size	Fineness Modulus	Per Cent Coarser than Each Sieve									
		100	48	28	14	8	4	⅜	¾	1½	2
0-1¼................	4.30	89	82	72	62	51	38	25	11	0	..
"	4.93	95	89	82	73	61	47	32	14	0	..
"	5.40	98	94	88	80	69	55	38	18	0	..
"	6.04	99	98	95	90	81	68	49	24	0	..
"	6.25	100	99	97	92	85	72	53	27	0	..
"	6.45	100	99	98	95	88	77	58	30	0	..
"	6.60	100	100	99	96	91	80	62	32	0	..
"	6.82	100	100	99	98	94	86	68	37	0	..

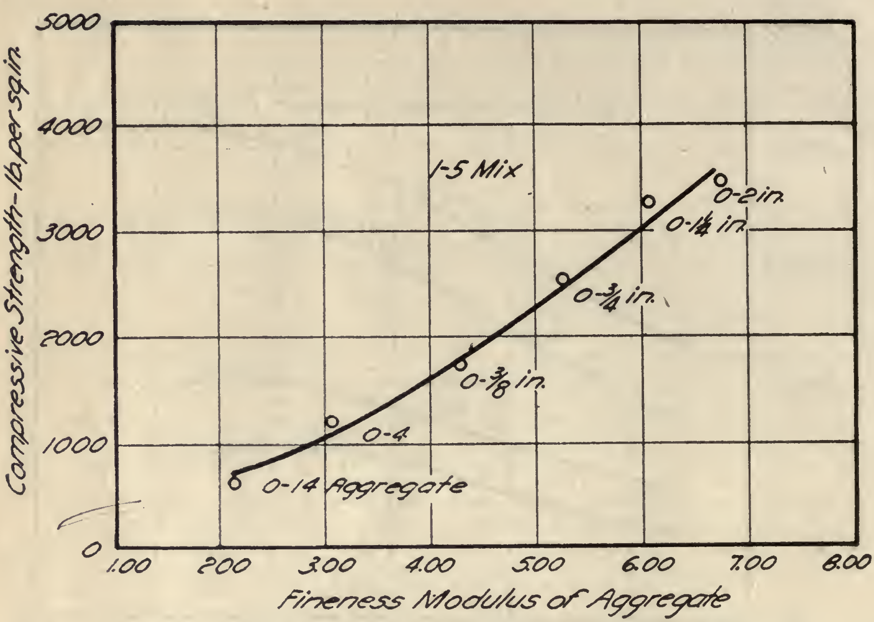

FIG. 4. RELATION BETWEEN FINENESS MODULUS OF AGGREGATE AND STRENGTH OF CONCRETE

Twenty-eight-day compression tests of 6 by 12-inch cylinders. (Series 78.) Sand and pebble aggregate graded to sizes shown. The contrast between the relation shown by these tests and those in Fig. 3 should be noted.

The sieve analyses of aggregates are given below:

Range in Size	Fineness Modulus	Per Cent Coarser than Each Sieve									
		100	48	28	14	8	4	⅜	¾	1½	2
0-14	2.16	95	84	37	0	..	..	..	..	..	..
0- 4	3.06	96	90	62	40	18	0	..	..	..	..
0- ⅜..............	4.26	96	91	83	71	54	31	0	..	..	..
0- ¾..............	5.24	98	96	91	83	71	54	31	0	..	..
0- 1¼..............	6.04	99	98	95	90	81	68	49	24	0	..
0- 2	6.72	100	99	97	94	87	77	62	42	14	0

Table 2

EFFECT OF GRADING OF AGGREGATES ON THE STRENGTH OF CONCRETE

Compression tests of 6 by 12-in. concrete cylinders.

Mix 1:5 by volume; age at test, 28 days; stored in damp sand; tested damp.

Aggregates—sand and pebbles from Elgin, Ill. Aggregates were screened to different sizes and recombined to conform to predetermined sieve analyses.

The aggregates were made up in such a manner as to give the widest variations in the grading of the particles. All gradings had one common property, in that the *fineness modulus* was exactly the same—m = 6.04. The fineness modulus of the aggregate is the sum of the percentages in the sieve analysis, divided by 100.

The same quantity of water was used in all specimens of a given consistency. The 110% consistency contains 10% more water than the 100%.

Each specimen was made from a separate batch.

Each value in the strength tests is the average from 5 tests made on different days. (From Series 78.)

Ref. No.	Sieve Analysis of Aggregate Per Cent Coarser than Each Sieve 100	48	28	14	8	4	3/8	3/4	1 1/4	1 1/2	2	Fineness Modulus of Aggregate	Surface Area of Aggregate sq. in. per lb. of Aggregate	per g. of Cement	Compressive Strength of Concrete at 28 days (lb. per sq. in.) 100% Consistency	110% Consistency
40	99	98	95	90	81	68	49	24	0			6.04	602	8.8	3,300	2,890
259	99	98	96	92	84	67	46	22	0			6.04	569	8.2	2,950	2,650
260	98	97	93	88	80	67	52	29	0			6.04	764	11.4	3,120	2,760
261	97	94	91	85	77	67	58	35	0			6.04	999	15.2	3,140	2,790
262	95	92	87	82	75	67	67	39	0			6.04	1,292	20.1	3,100	2,800
263	95	90	84	78	73	67	62	55	0			6.04	1,451	23.0	2,830	2,740
264	95	89	82	75	67	67	67	62	0			6.04	1,565	25.2	2,680	2,580
265	100	97	91	79	72	67	58	40	0			6.04	761	11.9	3,070	2,690
266	100	97	93	88	83	67	50	27	7	0		6.04	616	9.0	3,080	2,790
267	99	97	94	86	77	67	47	27	16		0	6.04	709	10.5	3,150	2,710
268	98	95	90	83	83	83	50	22	0			6.04	834	12.6	3,080	2,500
269	98	94	90	86	83	80	55	18	0			6.04	898	13.3	3,050	2,550
270	96	90	80	80	80	80	60	39	0			6.04	1,391	21.5	2,970	2,550
271	100	96	92	87	81	75	50	23	0			6.04	672	10.0	2,930	2,710
272	95	91	87	82	77	73	59	40	0			6.04	1,315	20.2	3,000	2,580
273	99	95	88	80	76	73	61	32	0			6.04	911	13.9	2,950	2,740
274	90	85	81	78	75	73	66	56	0			6.04	1,992	31.3	2,680	2,440
275	100	93	82	73	73	73	63	47	0			6.04	1,076	16.7	2,820	2,620
276	100	100	100	92	81	60	45	26	0			6.04	390	5.6	3,040	2,780
277	100	98	95	90	80	60	50	31	0			6.04	557	8.3	2,900	2,770
278	100	99	96	92	84	55	50	28	0			6.04	483	7.0	2,940	2,750
279	100	99	96	91	80	50	50	38	0			6.04	514	7.6	3,080	2,750
280	98	84	84	84	84	57	57	57	0			6.04	1,276	19.7	3,000	2,780
281	99	98	91	86	80	76	38	38	0			6.04	701	10.4	2,940	2,700
282	99	98	91	86	80	76	46	30	0			6.04	697	10.2	3,020	2,660
283	99	98	91	86	80	76	61	15	0			6.04	689	10.1	2,930	2,670
284	99	98	91	85	80	76	67	8	0			6.04	685	9.9	2,970	2,630
Average												6.04	904	13.8	2,990	2,690
Minimum Value												....	390	5.6	2,680	2,440
Maximum Value												....	1,992	31.3	3,300	2,890
Mean Variation from Average—per cent												....	34.4	37.2	3.41	3.04

the concrete as the fineness modulus of the aggregate increases, until a certain value is reached which corresponds to a maximum point. It will be noted also that this maximum point corresponds to higher and higher values of fineness modulus as the quantity of cement in the mix is increased. In other words, the maximum strength comes at a fineness modulus of about 5.80 for the 1:9 mix and about 6.40 for the 1:4 mix. In these tests the different values of the fineness modulus were secured by using a preponderance of the coarser sizes, but in all cases maintaining the same limiting size, that is, 1¼ in.

In Fig. 4 is found a similar relation between the strength and the fineness modulus, except that no maximum point is found. This condition arises from the fact that the maximum size of the aggregate is increasing without changing the type of the sieve analysis curve, consequently the fineness modulus strength curve continues to rise indefinitely. The height to which the curve rises is limited only by the maximum size of aggregate which may be used. It is important to note that there is no conflict between the indications of Figs. 3 and 4.

For all practical purposes and for ordinary ranges in concrete mixes the fineness modulus strength relation may be assumed as a linear one. The comparison of the true and approximate relation is brought out in the discussion of the "Water Formula" below.

A given value for the fineness modulus of an aggregate can be secured with any combination of percentages in the sieve analysis which gives the same total, consequently, an infinite variety of gradings may be found which give aggregate of the same concrete strength. Table 2 gives the results of groups of tests which bring out the wide variation which may be made in the grading of aggregate without producing any essential variation in the concrete strength. Twenty-seven different gradings of the same aggregate were made up. These gradings covered the widest possible range, but they had one property in common; that is, a fineness modulus of 6.04. All specimens were mixed with the same quantity of cement and water. Separate sets of specimens were made of two different consistencies. The mean variation from the average strength is about 3%.

Table 2 also furnishes some interesting data on the surface-area method of proportioning aggregates. It is seen that there is the widest variation in the surface area of the aggregate without any appreciable difference in the concrete strength. Our studies have clearly shown that surface area is not a satisfactory basis for proportioning aggregates.

Design of Concrete Mixes

In accordance with our previous statements the problem of designing concrete mixes using given materials resolves itself into that of finding the combination which with a given water-ratio will give a concrete of suitable workability using a minimum of cement.

The following outline will make clear the steps to be followed in the design of concrete mixes on the basis of our studies of concrete:

STEPS IN THE DESIGN OF CONCRETE MIXTURES

1. Knowing the compressive strength required of the concrete, determine by reference to Fig. 1 the maximum water-ratio which may be used. Subsequent steps in the design of concrete mixes are only devices for securing a workable concrete using this water-ratio and a minimum quantity of cement. It is obvious that a given water-ratio can be secured with a minimum of cement if the aggregate is graded as coarse as permissible (considering its size and the mix used) and if we use the driest mix which can be properly placed. Securing a coarse, well-graded aggregate, using rich mixes, employing the driest practicable consistency, using mechanical methods of placing concrete, etc., are all methods of producing a workable mix with a minimum water-ratio. Experience or trial is the only guide in determining the relative consistency of concrete necessary in the work. Obviously the driest workable consistency should be used.

The size of aggregate available, or which must be used, and the other factors will furnish a guide as to the mix. The mix is expressed as one volume of cement to a given number of volumes of aggregate; that is, the combined fine and coarse aggregate. In general, some allowance must be made for the high strengths in laboratory tests. In other words, a water-ratio somewhat lower than that given for the required strength in Fig. 1 should be used. For convenience in the subsequent steps we shall deal with concrete strength instead of water-ratio (as in Fig. 6), although it should be understood that it is the water-ratio which fixes the strength so long as we have a plastic mix.

2. Make sieve analysis of fine and coarse aggregates, using Tyler standard sieves of the following sizes: 100, 48, 28, 14, 8, 4, 3/8, 3/4 and 1½ in. Express sieve analysis in terms of percentages of material by weight (or separate volumes) *coarser than* each of the standard sieves.

3. Compute fineness modulus of each aggregate by adding the percentages found in (2), and dividing by 100.

4. Determine the "maximum size" of aggregate by applying the following rules: If more than 20% of aggregate is coarser than any sieve the maximum size shall be taken as the next larger sieve in the standard set; if between 11 and 20% is coarser than any sieve, maximum size shall be the next larger "half-sieve"; if less than 10% is coarser than certain sieves, the smallest of these sieve sizes shall be considered the maximum size.

5. From Table 3 determine the maximum value of fineness modulus which may be used for the mix, kind and size of aggregate, and the work under consideration. (The values in Table 3 are platted in Fig. 5.)

6. Compute the percentages of fine and coarse aggregates required to produce the fineness modulus desired for the final aggregate mixture by applying the formula:

$$p = 100 \frac{A - B}{A - C} \quad \ldots\ldots (3)$$

where p = percentage of fine aggregate in total mixture.
A = fineness modulus of coarse aggregate.
B = fineness modulus of final aggregate mixture.
C = fineness modulus of fine aggregate.

Fig. 7 may be used for solving Equation 3, and for making comparisons of the effect of certain changes in proportions of fine and coarse aggregates. The distinction between fine and coarse aggregate is solely for convenience in securing a uniform grading; the division may be made at any desired point.

7. With the estimated mix, fineness modulus and consistency enter Fig. 6 and determine the strength of concrete produced by the combination. If the strength shown by the diagram is not that required, the necessary readjustment may be made by changing the mix, consistency or size and grading of the aggregates.

The quantity of water required can be determined from Formula 4 below, or approximately from Table 5.

IMPORTANT NOTE: It must be understood that the values in Fig. 6 were determined from compression tests of 6 by 12-in. cylinders stored for 28 days in a damp place. The values obtained on the work will depend on such factors as the consistency of the concrete, quality of the cement, methods of mixing, handling, placing the concrete, etc., and on age and curing conditions.

Strength values higher than given for relative consistency of 1.10 should seldom be considered in designing, since it is only in exceptional cases that a consistency drier than this can be satisfactorily placed. For wetter concrete much lower strengths must be considered.

Table 3

MAXIMUM PERMISSIBLE VALUES OF FINENESS MODULUS OF AGGREGATES

For *mixes* other than those given in the table, use the values for the next leaner mix.

For *maximum sizes* of aggregate other than those given in the table, use the values for the next smaller size.

Fine aggregate includes all material finer than No. 4 sieve; *coarse aggregate* includes all material coarser than the No. 4 sieve. *Mortar* is a mixture of cement, water and fine aggregate.

This table is based on the requirements for *sand-and-pebble* or *gravel* aggregate composed of approximately spherical particles, in ordinary uses of concrete in reinforced concrete structures. For other materials and in other classes of work the maximum permissible values of fineness modulus for an aggregate of a given size is subject to the following corrections:

(1) If *crushed stone* or *slag* is used as coarse aggregate, *reduce* values in table by 0.25. For crushed material consisting of unusually flat or elongated particles, *reduce* values by 0.40.

(2) For *pebbles* consisting of *flat particles, reduce* values by 0.25.

(3) If *stone screenings* are used as fine aggregate, *reduce* values by 0.25.

(4) For the top course in *concrete roads, reduce* the values by 0.25. If finishing is done by *mechanical means,* this reduction need not be made.

(5) In work of *massive proportions,* such that the smallest dimension is larger than 10 times the maximum size of the coarse aggregate, *additions may be made* to the values in the table as follows: for ¾-in. aggregate 0.10; for 1½-in. 0.20; for 3-in. 0.30; for 6-in. 0.40.

Sand with fineness modulus lower than 1.50 is undesirable as a fine aggregate in ordinary concrete mixes. Natural sands of such fineness are seldom found.

Sand or screenings used for fine aggregate in concrete must not have a higher fineness modulus than that permitted for mortars of the same mix. Mortar mixes are covered by the table and by (3) above.

Crushed stone mixed with both finer sand and coarser pebbles requires no reduction in fineness modulus provided the quantity of crushed stone is less than 30% of the total volume of the aggregate.

Mix	Size of Aggregate													
Cem.-Agg.	0-28	0-14	0-8	0-4	0-3*	0-⅜	0-½*	0-¾	0-1 in.*	0-1½	0-2.1*	0-3 in.	0-4½*	0-6 in.
1-12	1.20	1.80	2.40	2.95	3.35	3.80	4.20	4.60	5.00	5.35	5.75	6.20	6.60	7.00
1-9	1.30	1.85	2.45	3.05	3.45	3.85	4.25	4.65	5.00	5.40	5.80	6.25	6.65	7.05
1-7	1.40	1.95	2.55	3.20	3.55	3.95	4.35	4.75	5.15	5.55	5.95	6.40	6.80	7.20
1-6	1.50	2.05	2.65	3.30	3.65	4.05	4.45	4.85	5.25	5.65	6.05	6.50	6.90	7.30
1-5	1.60	2.15	2.75	3.45	3.80	4.20	4.60	5.00	5.40	5.80	6.20	6.60	7.00	7.45
1-4	1.70	2.30	2.90	3.60	4.00	4.40	4.80	5.20	5.60	6.00	6.40	6.85	7.25	7.65
1-3	1.85	2.50	3.10	3.90	4.30	4.70	5.10	5.50	5.90	6.30	6.70	7.15	7.55	8.00
1-2	2.00	2.70	3.40	4.20	4.60	5.05	5.45	5.90	6.30	6.70	7.10	7.55	7.95	8.40
1-1	2.25	3.00	3.80	4.75	5.25	5.60	6.05	6.50	6.90	7.35	7.75	8.20	8.65	9.10

*Considered as "half-size" sieves; not used in computing fineness modulus.

Calculation of Water Required for Concrete

Because of the important influence of the quantity of water in the concrete it is desirable to have a sound basis for proportioning the water. The quantity of water necessary for given proportions and conditions may be determined by the following formula:

$$x = R\left[\frac{3}{2}p + \left(\frac{.30}{1.26^{m}} + a - c\right)n\right] \dots\dots\dots\dots (4)$$

where x = water required—ratio to volume of cement in batch (water-ratio).

R = Relative consistency of concrete, or "workability factor". Normal consistency (relative consistency = 1.00) requires the use of such a quantity of mixing water as will cause a slump of ½ to 1 in. in a freshly molded 6 by 12-in. cylinder of about 1:4 mix upon withdrawing the form by a steady, upward pull. A relative consistency of 1.10 requires the use of 10% more water and under the above conditions will give a slump of about 5 to 6 in.

p = Normal consistency of cement, ratio by weight.

m = Fineness modulus of aggregate (an exponent).

n = Volumes of mixed aggregate to one of cement.

a = Absorption of aggregate, ratio of water absorbed to volume of aggregate. (Determined after immersion in water for 3 hours. Average values for crushed limestone and pebbles may be assumed as 0.02; porous sandstones may reach 0.08; very light and porous aggregate may reach 0.25.)

c = Moisture contained in aggregate, ratio of water contained to volume of aggregate. (Assume as zero for room-dry aggregate.)

This formula takes account of all the factors which affect the quantity of water required in a concrete mixture. These factors may be classified as follows:

1. "Workability" factor, or the relative consistency of the concrete. This is dictated by the kind of work being done; concrete must be more plastic (which generally means a wetter consistency) in reinforced concrete building construction than is necessary in mass work. The term (R) in the equation takes care of this factor. (R) may vary from, say, 0.90 for a dry concrete to 2.00 or higher for very wet mixes.

2. Cement factor, which is made up of two parts: the *quality* of cement so far as normal consistency is concerned (p); the *quantity* of cement in the mix (n).

3. The aggregate factor. This includes the three terms within the parenthesis in Equation 4. The first term, involving (m), takes account of the size and grading; the second (a) the absorption, and the third (c) the water contained in the aggregate.

In case admixtures of any kind are used, another term must be inserted in the equation. This relation has been fully worked out, but is not included in this report.

Simplified Water Formula

While Equation (4) represents the true water relation, it is somewhat complicated by the fact that the fineness modulus (m) appears as an exponent. The equation can be expressed in a simpler form as follows:

$$x = R\left[\frac{3}{2}p + (0.22 - \frac{m}{42} + a - c)\,n\right] \dots\dots\dots (5)$$

This equation gives values for ordinary ranges of mix and grading of aggregate which are sensibly the same as given by Equation (4).

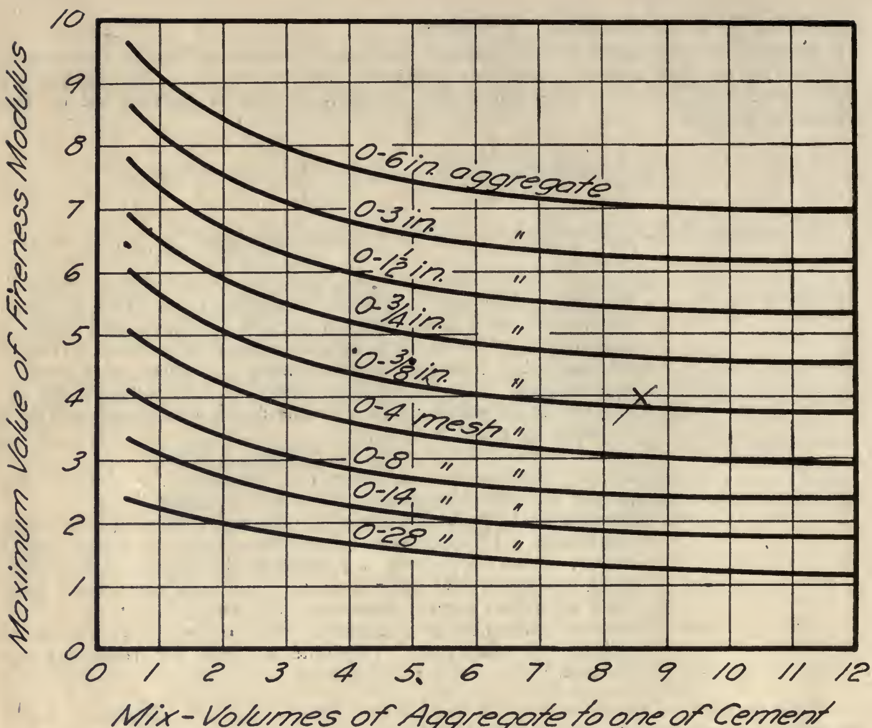

FIG. 5. MAXIMUM PERMISSIBLE VALUES OF FINENESS MODULUS OF AGGREGATE

Graphical reproduction of Table 3. These curves are based on the requirements of sand and pebble aggregate. For crushed stone aggregate the values must be reduced as noted in the table.

Maximum Permissible Values of Fineness Modulus of Aggregates

Since a maximum practicable value of fineness modulus is found for each size of aggregate and mix, it is necessary to place certain limits on the value which may be used for proportioning materials for concrete mixes. Table 3 gives limits which will be found practicable. Subsequent experience may dictate certain modifications in the details.

The purpose of Table 3 is to avoid the attempt to secure an aggregate grading which is too coarse for its maximum size and for the amount of cement used. It is also useful in prohibiting attempts to use sands which are too coarse for best results in concrete mixtures. For instance, it would be found from this table that the use of a sand of the nature of standard Ottawa sand is not permitted except in mixes 1:2 or richer.

The curves in Figure 5 are platted directly from the values given for the standard sieves in Table 3.

Chart for Design of Concrete Mixes

Fig. 6 is a nomographic chart for the design of concrete mixes. This chart takes account of the following four factors:

1. The mix (cement content).
2. The relative consistency.
3. The grading of aggregate (fineness modulus)
4. The compressive strength of concrete.

Given any three of these factors the chart enables us to solve for the fourth. This chart is, of course, based on the results of certain tests. For practical application these values must generally be reduced by certain factors, which will depend on the judgment of the designer. In order to furnish some basis for comparison, compression tests of 1:3 standard sand mortars from the cement used in these tests are given.

Suppose we consider the case of concrete for road construction. This is generally specified as a 1:1½:3 or a 1:2:3 mix, with aggregate graded up to 1½ in. These mixes are about the same as what have been termed a 1:4 mix, the exact equivalent depending on the particular size and grading of the fine and coarse aggregate. Assume that gravel aggregate will be used, graded to 1½ in. Table 3 shows that we may use a fineness modulus as high as 6.00 — .25 = 5.75. Knowing the sieve analysis and fineness modulus of both sizes of aggregate, apply the formula or Fig. 7 to determine the proportions of each aggregate which must be mixed to secure this value. Assume that the concrete will be mixed to a relative consistency of 1.10, which is of such plasticity as will give a slump of 5 to 6 in. in the test described above. Place a straightedge in Fig. 6 on mix 1:4 and fineness modulus 5.75, and mark the point where it crosses the reference line for consistency; from this point project the line horizontally (as indicated in other examples) to relative consistency 1.10. It will be seen that this gives a compressive strength of 3,400 lb. per sq. in. at 28 days.

Table 4

EXAMPLE OF INFLUENCE OF QUANTITY OF MIXING WATER ON THE STRENGTH OF CONCRETE

Values calculated from equation

$$S = \frac{A}{B^x} = \frac{14{,}000}{8.2^x}$$

Where S = Compressive strength of concrete (lb. per sq. in.).
x = Water-ratio (an exponent).

A and *B* are constants whose values depend on quantity of cement and other conditions of the test. The values given for *A* and *B* are based on 28-day tests of 1:4 mix, pebble aggregate graded 0-1¼-in., fineness modulus 5.75.

The water-ratio is equivalent to the cubic feet of water to 1 sack (1 cu. ft.) of cement.

The strength values are solely for comparative purposes in showing the influence of changing the water content.

Water in a 1-Bag Batch		Relative Consistency	Compressive Strength of Concrete at 28 Days	
			Lb. per Sq. In.	Relative Strength
Gallons	Water-Ratio (x)	Per Cent	(S)	Per Cent
5.75	.77	100	2,770	100
6.0	.80	104	2,600	94
6.25	.84	109	2,400	87
6.5	.87	113	2,250	81
7.0	.94	122	1,950	70
7.5	1.01	131	1,670	60
8.0	1.07	139	1,470	53
9.0	1.21	157	1,100	40
10.0	1.34	174	830	30
12.0	1.60	208	480	17
15.0	2.00	260	200	7

Table 5

QUANTITY OF MIXING WATER REQUIRED FOR CONCRETE

Calculated by formula: $$x = R\left[\frac{3}{2}p + \left(\frac{.30}{1.26^{m}} + a - c\right)n\right]$$

Where x = Water required—ratio to volume of cement in batch (water-ratio).

R = Relative consistency, or "workability factor." Where R = 1.00 the concrete is said to be of "normal consistency."

p = Normal consistency of cement by weight (assume p = 0.23).

m = Fineness modulus of aggregate.

n = Volume of mixed aggregate to one volume of cement.

a = Absorption of aggregate, ratio of water absorbed to volume of aggregate.

c = Moisture in aggregate, ratio of water contained to volume of aggregate.

(a — c) = Net absorption of aggregate by volume.

In this table (a — c) is assumed as 0.02. In other words the net quantity of water taken by the aggregate is 2% by volume. This value may be used for ordinary limestones and pebbles. For crushed trap and granite it is somewhat high. It is too high in any case where the aggregate is saturated with water.

A relative consistency of 1.00 (normal consistency) requires the use of such a quantity of mixing water as will cause a slump of ½ to 1 in. in a freshly molded 6 by 12-in. cylinder of about 1:4 mix upon withdrawing the form by a steady, upward pull. This consistency is somewhat dry for most concrete work, but can be used where light tamping is practicable.

A relative consistency of 1.10 (10% more water than required for normal consistency) represents about the driest concrete which can be satisfactorily used in concrete road construction. Under the conditions mentioned above, this consistency will give a slump of about 5 to 6 in.

A relative consistency of 1.25 represents about the wettest consistency which should be used in reinforced concrete building construction. Under the conditions mentioned above, this consistency will give a slump of about 8 to 9 in.

For mixes and fineness moduli, other than those given in the table, approximate values may be determined by interpolation. For specific cases use the formula.

Mix Cem.-Agg. by Volume	Gallons of Water per Sack of Cement Using Aggregates of Different Fineness Moduli											
	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00
	Relative Consistency — (R) = 1.00											
1-12........	23.5	21.4	19.5	17.8	16.4	15.2	13.9	12.9	12.0	11.1	10.4	9.8
1-9........	18.1	16.7	15.2	14.0	12.9	12.0	11.0	10.2	9.6	9.0	8.4	7.9
1-7........	14.7	13.5	12.3	11.4	10.6	9.9	9.1	8.6	8.0	7.6	7.2	6.7
1-6........	13.0	12.0	11.0	10.2	9.5	8.9	8.3	7.7	7.3	6.8	6.5	6.2
1-5........	11.2	10.4	9.5	8.9	8.3	7.8	7.3	6.9	6.4	6.1	5.8	5.5
1-4........	9.5	8.9	8.2	7.7	7.2	6.8	6.3	6.0	5.7	5.4	5.2	5.0
1-3........	7.8	7.2	6.7	6.3	6.0	5.7	5.4	5.1	4.9	4.6	4.5	4.3
1-2........	6.0	5.7	5.4	5.1	4.9	4.7	4.5	4.3	4.1	4.0	3.9	3.8
1-1........	4.3	4.1	3.9	3.8	3.7	3.6	3.5	3.4	3.3	3.2	3.2	3.1
	Relative Consistency — (R) = 1.10											
1-12........	25.8	23.6	21.4	19.6	18.1	16.7	15.3	14.2	13.2	12.2	11.4	10.8
1-9........	19.9	18.4	16.7	15.4	14.2	13.2	12.1	11.2	10.6	9.9	9.2	8.7
1-7........	16.2	14.9	13.5	12.5	11.7	10.9	10.0	9.5	8.8	8.4	7.9	7.4
1-6........	14.3	13.2	12.1	11.2	10.5	9.8	9.1	8.5	8.0	7.5	7.2	6.8
1-5........	12.3	11.4	10.5	9.8	9.1	8.6	8.0	7.6	7.0	6.7	6.4	6.1
1-4........	10.5	9.8	9.0	8.5	7.9	7.5	6.9	6.6	6.3	5.9	5.7	5.5
1-3........	8.6	7.9	7.4	6.9	6.6	6.3	5.9	5.6	5.4	5.1	5.0	4.7
1-2........	6.6	6.3	5.9	5.6	5.4	5.2	5.0	4.7	4.5	4.4	4.3	4.2
1-1........	4.7	4.5	4.3	4.2	4.1	4.0	3.9	3.7	3.6	3.5	3.5	3.4
	Relative Consistency — (R) = 1.25											
1-12........	29.4	26.8	24.4	22.2	20.5	19.0	17.4	16.1	15.0	13.9	13.0	12.3
1-9........	22.6	20.9	19.0	17.5	16.1	15.0	13.8	12.7	12.0	11.2	10.5	9.9
1-7........	18.4	16.9	15.4	14.3	13.2	12.4	11.4	10.7	10.0	9.5	9.0	8.4
1-6........	16.3	15.0	13.8	12.8	11.9	11.1	10.4	9.6	9.1	8.5	8.1	7.7
1-5........	14.0	13.0	11.9	11.1	10.4	9.8	9.1	8.6	8.0	7.6	7.2	6.9
1-4........	11.9	11.1	10.2	9.6	9.0	8.5	7.9	7.5	7.1	6.8	6.5	6.2
1-3........	9.8	9.0	8.4	7.9	7.5	7.1	6.8	6.4	6.1	5.8	5.6	5.4
1-2........	7.5	7.1	6.8	6.4	6.1	5.9	5.6	5.4	5.1	5.0	4.9	4.8
1-1........	5.4	5.1	4.9	4.8	4.6	4.5	4.4	4.3	4.1	4.0	4.0	3.9

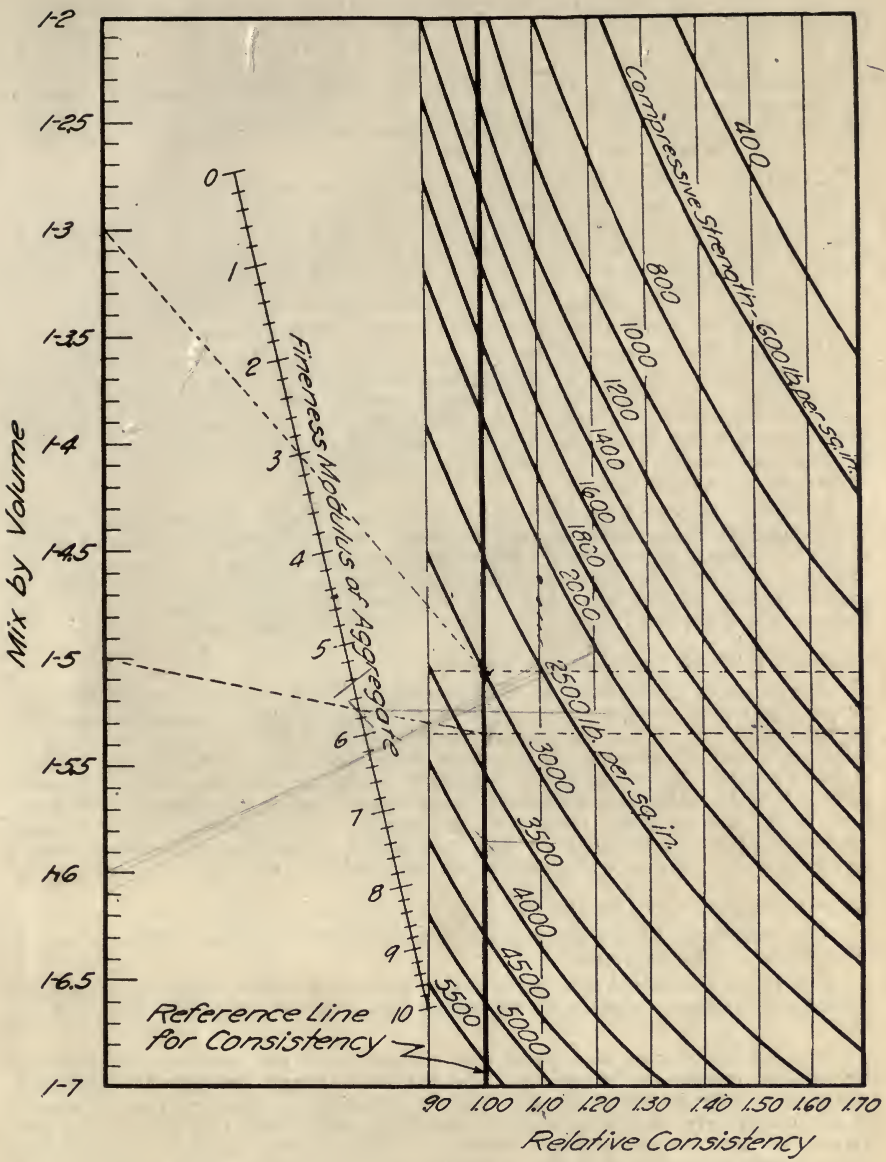

FIG. 6. DIAGRAM FOR THE DESIGN OF CONCRETE MIXTURES

This chart is based on compression tests of 6 by 12-inch cylinders; age 28 days; stored in damp sand. The cement used gave compressive strengths in 1-3 standard sand mortar as follows:

Age	Lb. per Sq. In.
7 days	1,900
28 days	3,200
3 months	4,200
1 year	4,300

The effect of using other mixes, gradings or consistencies on the strength can be seen at once from the diagram. For instance, if the water were increased to a relative consistency of 1.25 (not nearly so wet as is frequently seen in road work) the strength will be reduced to 2,700 lb. per sq. in.—a reduction of over 20 per cent. If the mix were changed to 1:4½ and other factors the same as in the first example, the strength would be 3,200 lb. per sq. in. We should have to change the mix to as lean as 1:5¼ in order to secure the same reduction in strength as was found above for a change from 1.10 to 1.25 consistency.

By using the wetter of the two consistencies we secure concrete of the same strength as if we had used one-third less cement and the drier mix. In other words, increasing the mixing water 13% causes the same reduction in strength as if we should omit 33% of the cement. This example shows the reason for emphasizing the importance of proper control of mixing water in concrete.

This chart enables us to answer such questions as the following:

Which is the stronger, a 1:3 mortar or a 1:5 concrete mixture?

Assuming that concrete of the same plasticity is used, the relative strengths will depend, of course, on the grading of the aggregates and the mix. In one case we have assumed 1:3 mix with fineness modulus equal to 3.00. This will give a strength for normal consistency of 3,000 lb. per sq. in. The 1:5 mix (fineness modulus 5.70) gives a strength for normal consistency of about 3,300 lb. per sq. in. The strengths for other consistencies can be found by reading horizontally across the chart as indicated by the dotted lines.

Unfortunately, we now have no proper basis for absolute values for strength of concrete. This, of course, makes it necessary to refer to particular tests as in Fig. 6. This condition emphasizes the importance of working out a test of cement which will give us at once the concrete strength for given materials, mixes, etc. With the present method of testing cement it is impossible to do more than make a rough guess as to the strength of concrete from the results of briquet tests.

Quantity of Water Required for Concrete

The formulas given above (4 and 5) show the elements which make up the water-requirements of a concrete mix. Table 5 gives the quantity of water required for certain mixes and values of fineness modulus. Quantities are given in terms of gallons per sack of cement. In this table the net absorption (that is, the quantity of water taken up by the aggregate in addition to that already contained) is assumed as 0.02 (2% by volume). This table is of interest when we consider that it has been found that a given water-ratio corresponds to constant concrete strength regardless of the combination of mix, consistency or grading of aggregate which may be used, so long as we have a workable concrete.

Further Discussion of Concrete Mixes

The importance of the water-ratio on the strength of concrete will be shown in the following considerations:

One pint more water than necessary to produce a plastic concrete reduces the strength to the same extent as if we should omit 2 to 3 lb. of cement from a 1-bag batch.

Our studies give us an entirely new conception of the function performed by the various constituent materials. The use of a coarse, well-graded aggregate results in no gain in strength unless we take advantage of the fact that the amount of water necessary to produce a plastic mix can thus be reduced. In a similar way we may say that the use of more cement in a batch does not produce any beneficial effect except from the fact that a plastic, workable mix can be produced with a lower water-ratio.

The reason a rich mixture gives a higher strength than a lean one is not that more cement is used, but because the concrete can be mixed (and usually is mixed) with a water-ratio which is relatively lower for the richer mixtures than for the lean ones. If advantage is not taken of the fact that in a rich mix relatively less water can be used, no benefit will be gained as compared with a leaner mix. In all this discussion the quantity of water is compared with the quantity of cement in the batch (cubic feet of water to 1 sack of cement) and not to the weight of dry materials or of the concrete as is generally done.

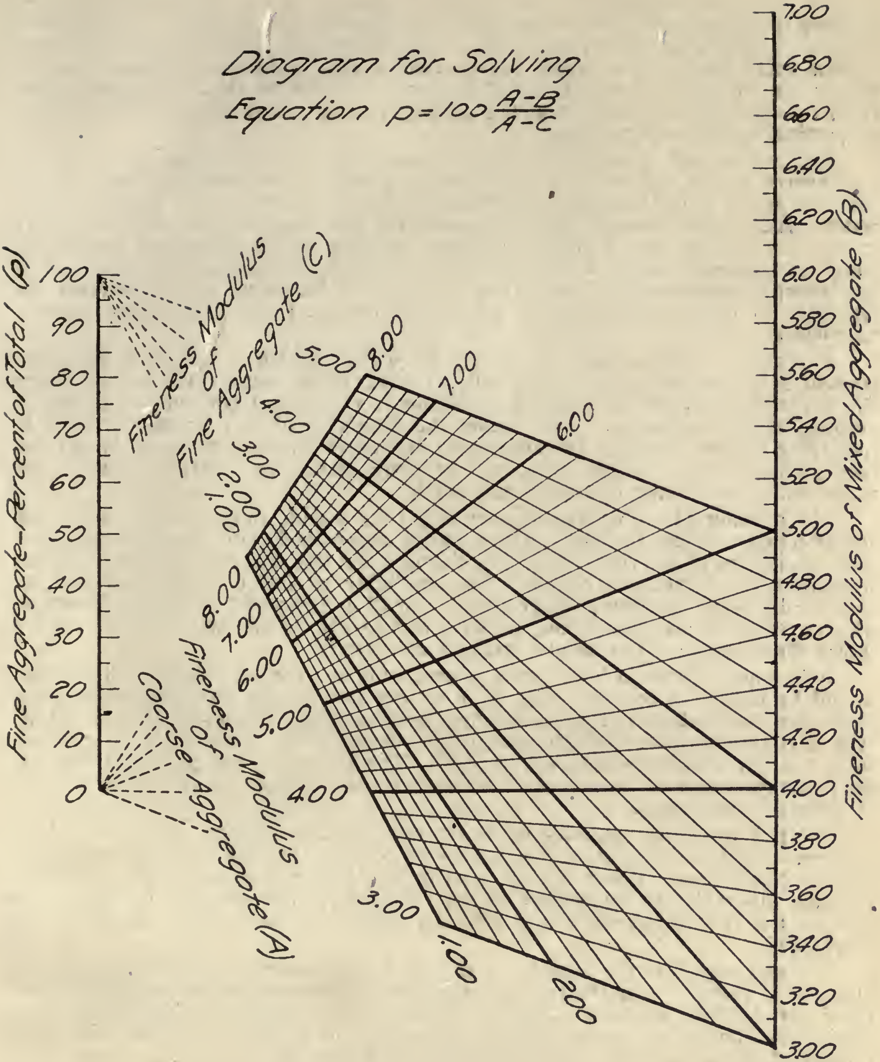

FIG. 7. DIAGRAM FOR DETERMINING QUANTITY OF SAND REQUIRED IN CONCRETE MIXES

Based on equation $p = 100 \frac{A-B}{A-C}$

where p = percentage of fine aggregate in total mixture.
A = fineness modulus of coarse aggregate.
B = fineness modulus of total aggregate.
C = fineness modulus of fine aggregate.

The mere use of richer mixes has encouraged a feeling of security, whereas in many instances nothing more has been accomplished than wasting a large quantity of cement, due to the use of an excess of mixing water. The universal acceptance of this false theory of concrete has exerted a most pernicious influence on the proper use of concrete materials and has proven to be an almost insurmountable barrier in the way of progress in the development of sound principles of concrete proportioning and construction.

Rich mixes and well-graded aggregates are just as essential as ever, but we now have a proper appreciation of the true function of the constituent materials in concrete and a more thorough understanding of the injurious effect of too much water. Rich mixes and well-graded aggregates are after all only a means to an end; that is, to produce a plastic, workable concrete with a minimum quantity of water as compared with the cement used. Workability of concrete mixes is of fundamental significance. This factor is the only limitation which prevents the reduction of cement and water in the batch to much lower limits than are now practicable.

The above considerations show that the water content is the most important element of a concrete mix, in that small variations in the water cause a much wider change in the strength than similar variations in the cement content or the size or grading of the aggregate. This shows the absurdity of our present practice in specifying definite gradings for aggregates and carefully proportioning the cement, then guessing at the water. It would be more correct to carefully measure the water and guess at the cement in the batch.

The grading of the aggregate may vary over a wide range without producing any effect on concrete strength, so long as the cement and water remain unchanged. The consistency of the concrete will be changed, but this will not affect the concrete strength if all mixes are plastic. The possibility of improving the strength of concrete by better grading of aggregates is small as compared with the advantages which may be reaped from using as dry a mix as can be properly placed. Table 4 shows the effect of water on the strength of concrete.

It is impracticable to lay down a general rule for the quantity of water which should be used in a concrete mix, since it was seen in the water formulas given above that the total water is governed by a large number of different factors. However, it is only the water which goes to the cement (that is, exclusive of absorbed water) which affects the concrete strength. The failure to recognize this fact has led to many erroneous conclusions from tests made to determine the relative merits of different aggregates.

Table 5 gives the quantity of water required for plastic mixes for certain assumed conditions of normal consistency of cement, absorption of aggregate, and relative consistency. Water is expressed in terms of gallons per sack of cement. In using this table the dependence of the value of fineness modulus which may be used on the size of aggregate and the mix, referred to in Table 3, should not be overlooked.

Without regard to the actual quantity of mixing water, the following rule is a safe one to follow: *Use the smallest quantity of mixing water that will produce a plastic or workable concrete.* The importance of any method of mixing, handling, placing and finishing concrete which will enable the builder to reduce the water content of the concrete to a minimum is at once apparent.